United States
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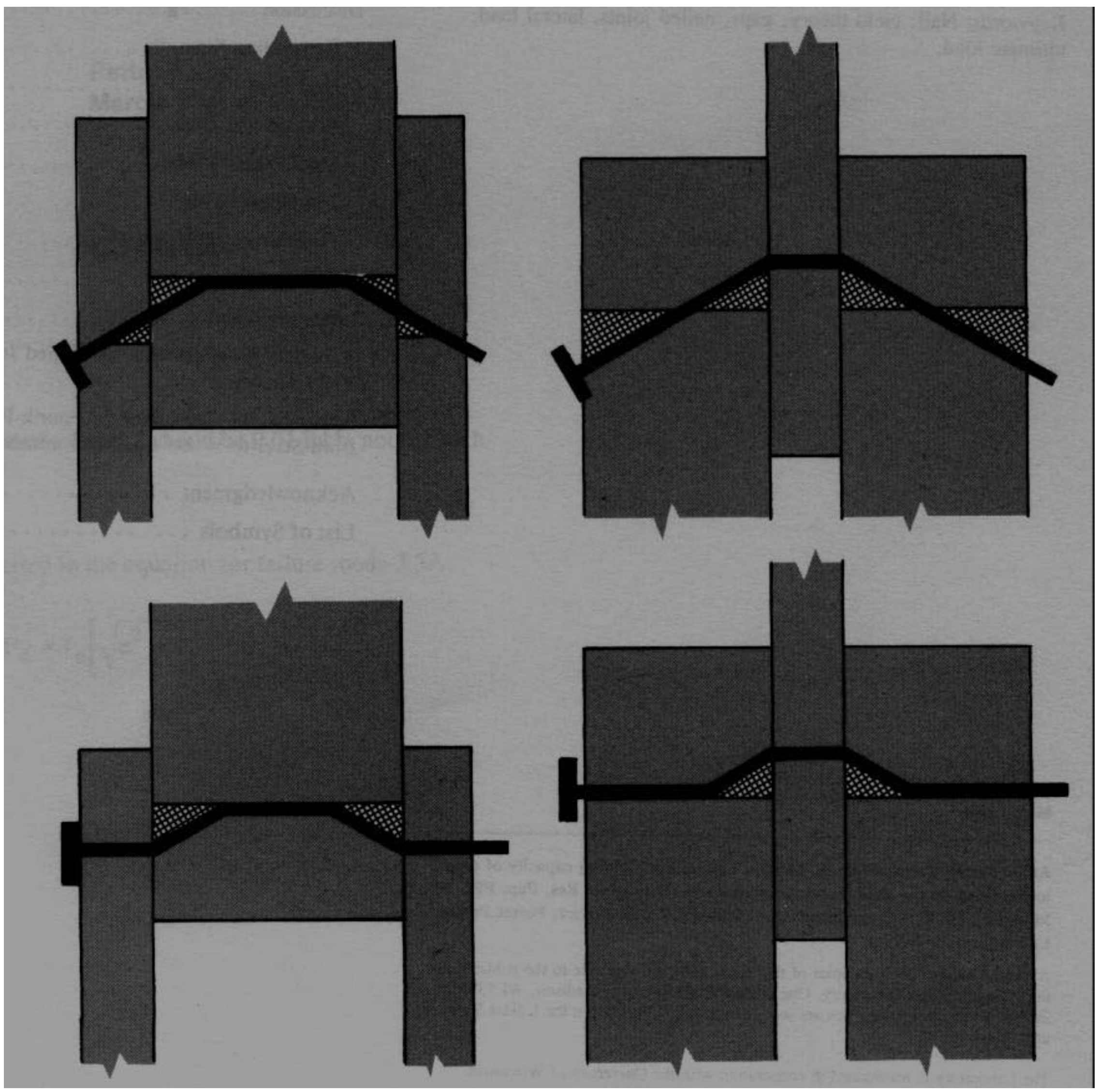
FPL 470

Lateral Load-Bearing Capacity of Nailed Joints Based on the Yield Theory

Experimental Verification

Petter Aune
Marcia Patton-Mallory



Abstract

The yield theory of joint loading provides a method to analyze nailed joints using a material science approach. The analysis requires a knowledge of two material properties, wood embedding strength and nail yield moment. The yield theory predicts the ultimate lateral load for bolted or nailed joints. The purpose of this study is to verify the yield theory experimentally for a limited number of nailed joints using wood species and joint geometries typical of construction in the United States. The yield theory is applied to a variety of nailed joint configurations. The strengths of 265 nailed joints subject to short-term loading are predicted on the basis of nail and wood properties and confirmed by experiments. The experimental tests validate application of the yield theory also to nailed joints whose members have dissimilar wood properties, to joints with steel center plates or with a gap caused by insulation between joint members. Finally, the stiffness and strength of two-member joints measured by American test method (ASTM) are compared to those of three-member joints measured by European test method.

Keywords: Nail, yield theory, gaps, nailed joints, lateral load, ultimate load.

March 1986

Aune, Petter; Patton-Mallory, Marcia. Lateral load-bearing capacity of nailed joints based on the yield theory: Experimental verification. Res. Pap. FPL 470. Madison, WI. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory; 1986. 29 p.

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Errata

Page 12, Table 5

Deformation δ for columns 1 and 2 should be 0.01 inch, not 0.1 inch.

Page 27, Table A7

Brackets should be inserted in the equation for failure mode 3.3A:

$$F_u = f_e\left[\sqrt{e^2 + 2_\gamma}\right] - e$$

November 1993

Lateral Load-Bearing Capacity of Nailed Joints Based on the Yield Theory
Experimental Verification

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Introduction

The ultimate lateral load of a nailed timber joint can be predicted using a theory of "yielding" which assumes plasticity in both the wood and the fastener; this we refer to as the yield theory. European researchers have verified the yield theory for bolted and nailed joints (Aune 1966, Johansen 1941, Mack 1960, Möller 1950, Siimes et al. 1954). However, this theory has only recently been verified for bolted joints using wood species and joint geometries typical of construction in the United States (McLain and Thangjitham 1983).

The main purpose of the research reported here was to verify the yield theory experimentally for a limited number of nailed joints using the wood species and joint geometries commonly found in the United States. We present the experimental results for nailed joints having wood members with similar and dissimilar embedding strengths, or with steel center plates, and for joints with members in contact or with gaps caused by insulation between joint members. We compare these results with theoretical predictions, showing how they confirm the validity of the yield theory. In the light of the experimental results, we consider extending the theory by substituting a fourth-root curve for the assumed plastic wood embedment relationship.

A secondary objective of this research was to compare the standard test method of the American Society for Testing and Materials (ASTM D 1761) to Nordtest method. We report observations of joint stiffness and strength to compare, first, two-member joints (ASTM D 1761) to three-member joints (Nordtest) and, second, joints with nailheads driven flush to the wood surface (ASTM) to joints with nailheads left above the wood surface (Nordtest). In interpreting these results, we consider the effect of nailhead fixity and of friction.

In addition to achieving these goals, we believe the test results may assist researchers and code-writing officials in understanding the behavior of nailed joints. The results of testing joints with a layer of insulation between wood members illustrate the general case of shear walls with insulation between sheathing and framing materials. Test results with nailed joints combining steel plates and wood members may be used to assess joint efficiency, or measure the sensitivity of yield load to the method of determining embedding strength. The use of a fourth-root wood embedding relationship may enable us to predict joint deformation using the yield theory.

The most important application we envision for this research is to promote the adoption of a uniform method of joint analysis, based on the yield theory which is applicable not only to nailed joints but to joints containing bolts or lag screws.

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[1] Visiting scientist at FPL, October 1981-June 1982.

General Yield Theory

The yield theory predicts a nailed joint's ultimate lateral load based on the embedding strength of the wood and the yield moment of the nail. This generalized yield theory for nailed joints is often referred to as "Möller's theory" (Möller 1950), although the principles were initially introduced by Johansen (1941) 10 years earlier. The historical development of the theory is summarized by Aune and Patton-Mallory (1986) in a complete discussion of the yield theory.

In the general yield theory both the yielding of the fastener and the embedding of the wood are assumed ideally plastic (fig. 1). The assumed curve of fastener bending moment versus angular rotation (fig. la) approximates the behavior of a nail quite well; the assumed curve of wood embedment load versus deformation (fig. lb) is less certain; however, research (Aune 1966) indicates that, as approximations, both assumed relationships are adequate to predict joint yield.

A number of failure modes are possible in a joint subject to lateral load (figs. 2, 3, 4). The failure mode is a function of wood embedment properties, nail yield moment, member thickness, and other joint geometries.

All failure modes involve either nail yield in bending, wood crushing (yielding), or some combination of the two. The ultimate lateral load (yield load) is determined by the failure mode.

The basic yield model predicts joint yield without considering joint deformation. The yield model assumes the joint does not fail at loads below joint yield because of insufficient spacing or end distances. Finally, the yield model ignores friction because it is difficult to estimate accurately, and in many joints does not exist.

Aune and Patton-Mallory (1986) have presented formulas for nailed joints based on the yield theory (also see Appendix A). The formulas provide yield load for wood-to-wood and steel-to-wood joints including joints with dissimilar embedment strength of wood members. Additional formulas apply when a layer of insulation (or gap) exists between joint members. A final formula represents the relationship between the wood's embedding stress and deformation as a fourth-root curve.

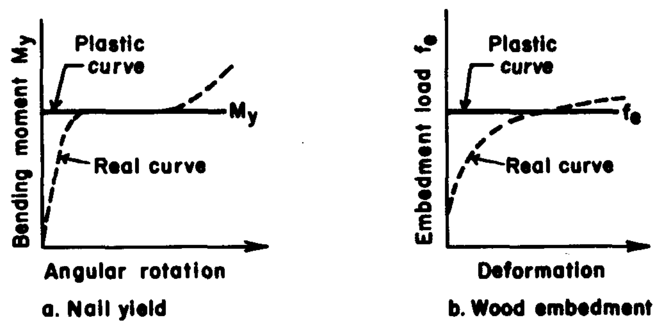

Figure 1 .—(a) Nail yield and (b) wood embedment both are assumed ideally plastic in the generalized yield theory. (ML85 5236)

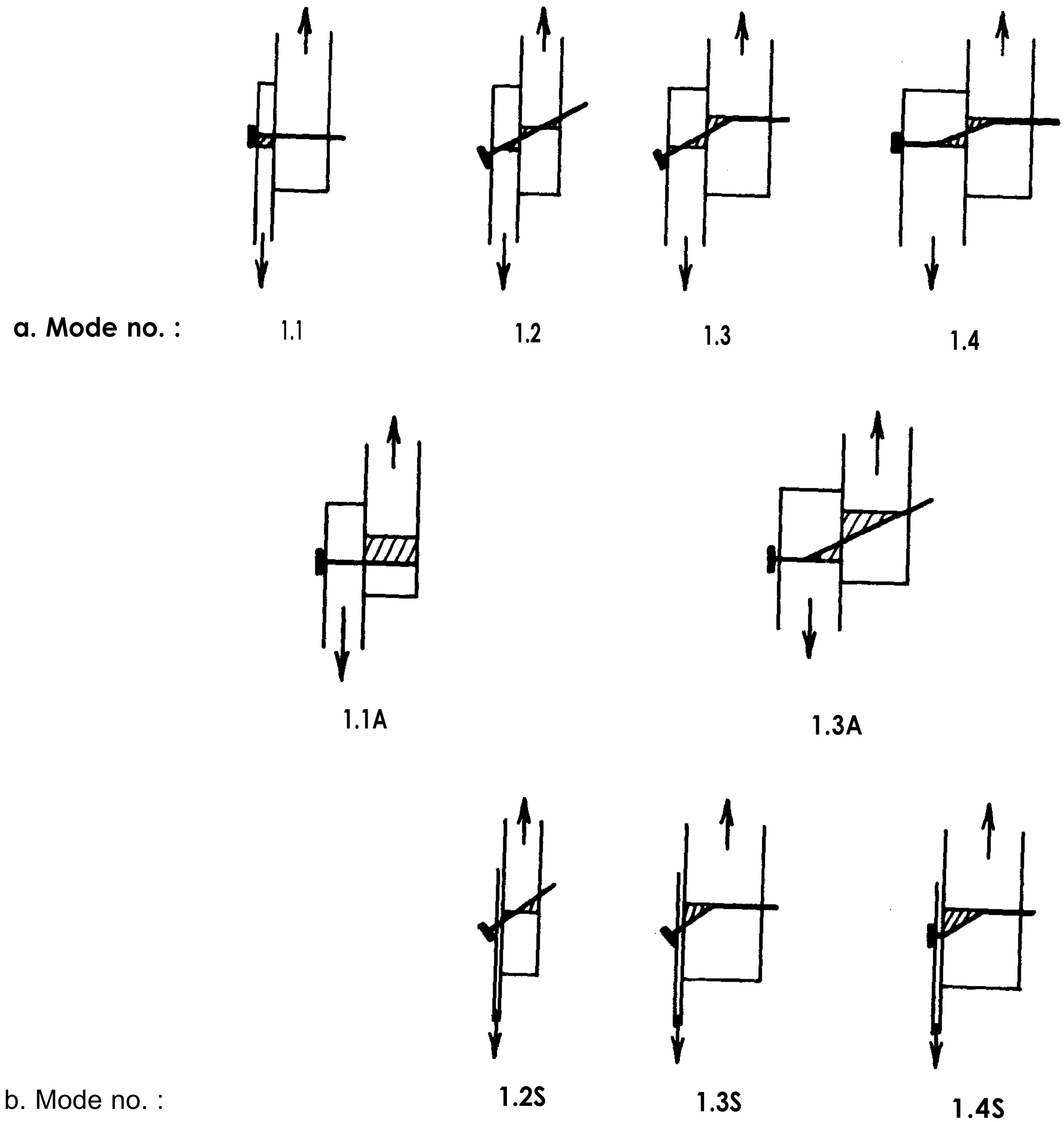

Figure 2.—Failure modes for two-member joints with (a) wood side member and (b) with steel side member. (ML85 5342)

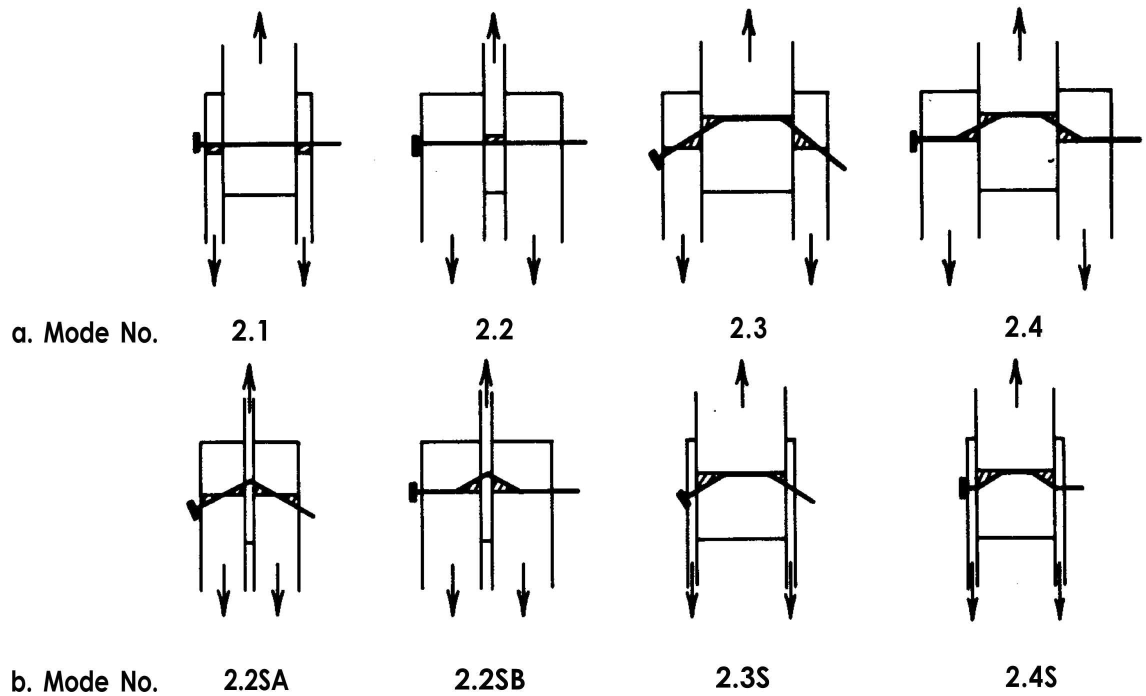

Figure 3.—Failure modes for three-member joints (a) with all wood members and (b) with steel side or main members. (ML85 5340)

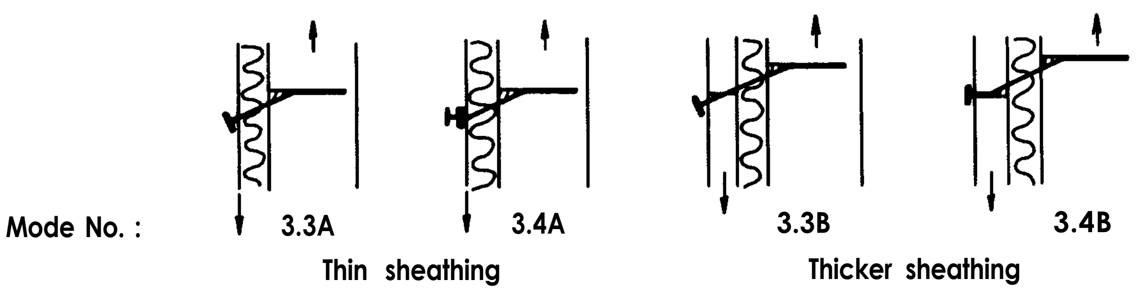

Figure 4.—Failure modes for two-member joints with a layer of insulation between wood members. (ML85 5341)

Materials and Methods

The experimental program (table 1) included 17 different joint configurations with 14 to 20 replications of each joint type. Table 1 lists joint member materials, member thicknesses, nail size and gap size, and a test group identification number for each joint type.

To isolate the effect of gaps and other variables, the same sets of wood members were used repeatedly within test groups 1-3, 4-7, 8-9, 10-11, and 12-13. Members were used only once for test groups 14-17. Shims were used during assembly of joints with an "air" gap indicated in table 1, and removed before testing. Other joints had the initial gap maintained during testing by a layer of polyethylene, polystyrene, or beadboard (see table 1 and fig. 4). Test group 17 was a steel-to-wood joint used to evaluate the yield model for joints with steel side plates.

Lumber and Plywood

Most test specimens were construction-grade Douglas-fir 2 by 4 lumber. The wooden joint members (one, two, or three for each test joint) were cut from 16-foot-long pieces and numbered consecutively. With few exceptions, no test group had more than one member from one particular piece of lumber.

Side members for test group 15 were spruce-pine-fir (SPF) of nominal dimensions 2 by 4 inches. The specimen members were cut from five 16-foot-long pieces, of which two were identified as spruce and three as fir.

Table 1.—Nailed joint test program

Materials				Nail Size	Fabrication				Test group No.
Side member		Center member			Number of nails	Gap		Replications	
Material	t_1, thickness	Material	t_2, thickness			Size	Material		
	In.		*In.*			*In.*			
SINGLE-SHEAR JOINTS									
Douglas-fir lumber	1.5	Douglas-fir lumber + plywood	1.5 + 0.5	40d	1	0.009	air	16	1
					2	0.036	air	16	2
					2	0.040	Polyethylene film	15	3
CDX 5-ply plywood	0.45	Douglas-fir lumber	1.5	8d	2	0.009	air	15	4
					2	0.036	air	15	5
					2	0.040	Polyethylene film	15	6
					[1]2	0.009	air	15	[1]7
CDX 5-ply plywood	0.45	Douglas-fir lumber	3.23	8d	[1]1	[2]0.0		20	8
					1	0.009	air	19	9
CDX 5-ply plywood	0.47	Douglas-fir lumber + plywood	1.5 + 0.5	40d	2	0.009	air	14	10
					2	0.97	Polystyrene	15	11
CDX 5-ply plywood	0.47	Douglas-fir lumber + plywood	1.5 + 0.5	40d	[3]2	0.97	Polystyrene	14	[3]12
					2	0.50	beadboard	14	13
DOUBLE-SHEAR JOINTS									
Douglas-fir lumber	1.5	Douglas-fir lumber	1.5	40d	2	0.009	air	15	14
Spruce-pine-fir lumber	1.5	Douglas-fir lumber	1.5	40d	2	0.009	air	15	15
CDX 5-ply plywood	0.45	Douglas-fir lumber	1.5	8d	4	0.009	air	15	16
Douglas-fir lumber	2.1	Steel plate	0.19	40d	1	~0.008	air	17	17

[1]Nailheads driven flush to member surface.

[2]Members in tight contact according to ASTM D 1761.

[3]Nailhead restrained by 1-in. ashboard sheathing.

Before the test specimens were assembled, the lumber was conditioned under controlled temperature of 73 °F and relative humidity of 50 percent. These conditions correspond to an equilibrium moisture content of about 11 percent. After testing, sections were cut from the specimens to determine moisture content and specific gravity (shown in table 2).

The plywood used in these tests was 1/2-inch-thick CD five-ply Exterior Plywood sheets (4 by 8 ft). The specimens were randomly selected from sheet 1 for test groups 4, 8, and 16 and from sheet 2 for test groups 11, 12, and 13.

Before assembly the plywood members were conditioned in controlled temperature and relative humidity as previously described for the lumber.

Nail Yield Moments

Common wire nails described in the Wood Handbook (Forest Products Laboratory 1974) were evaluated for yield moments. Testing included 10 replications of each nail size: 8d, 16d, and 40d. The 40d nails were purchased in two lots; the second lot was used only in one test group (group 17).

To obtain the nail yield moment M_y the nails were subjected to bending in a three-point loading setup (fig. 5). Span and rate of loading were adjusted for each nail size (see table 3). This test is similar but not identical to the Nordtest method (Nordtest 1981), in which the relationship between load and displacement of the crosshead is used to calculate the curve of bending moment versus angular rotation (fig. 10).

Embedding Strength

The wood embedding strength tests were conducted according to the Nordtest method (1981) using wood at a moisture content of approximately 10 percent. The wood specimens were 5.9 inches long and 3.5 inches wide.

The five embedding strength test groups involved 8d and 40d nails and Douglas-fir lumber, SPF lumber, and plywood (table 4). Each test group consisted of 15 replications. Nails rested in prebored holes equal to the hole diameters used to assemble nailed joints.

The specimens were loaded in compression by means of a jig shown in figure 6. The tests were conducted in a 10,000-pound hydraulic testing machine. The load was applied parallel to the grain (for plywood, parallel to the grain of the outer veneer) with the loading head of the machine moving at a constant speed of 0.05 inch per minute. The testing was stopped at a deflection of 0.25 inch for 40d nails and 0.15 inch for 8d nails. The load-deflection curves were recorded continuously, with the deflection assumed to be equal to the travel of the movable head of the testing machine.

Nailed Joint Tests

The primary purpose of the test program was to verify the yield theory's prediction of yield load for various nailed joints. The test program was also designed for the purpose of comparing test results obtained by use of two different test methods. Thus, the two-member tests in general were conducted according to ASTM D 1761-77 Standard Methods of Testing-Metal Fasteners in Wood (ASTM 1977) and the three-member tests according to the Nordtest method NT Build 133—Nails in Wood-Lateral Strength (Nordtest 1981).

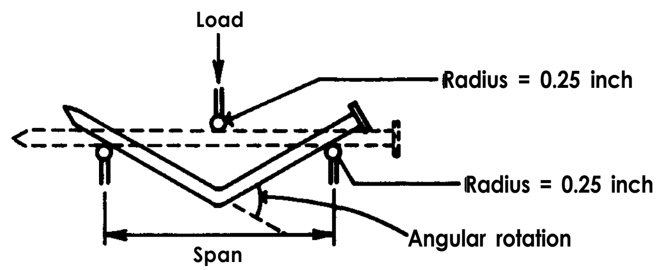

Figure 5.—Nail yield moment test. (ML85 5331)

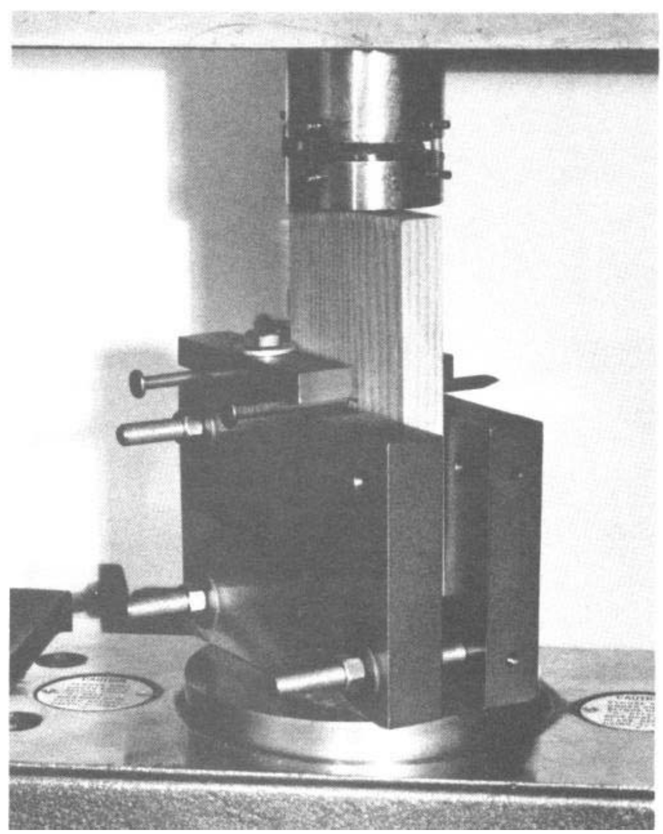

Figure 6.—Wood embedment test apparatus. Nail is driven into a prebored hole and is loaded in compression, recording load and crosshead deformation. (M 150804-6)

Table 2.—Moisture content and specific gravity of the lumber for all test groups (high, low, and mean values)

Wood members	Test Group No.	Number of tests	Moisture content			Specific gravity		
			High	Low	Mean	High	Low	Mean
			--------------Pct-------------					
Douglas-fir	1-3	15	11.2	9.6	10.1	0.59	0.39	0.46
	4-7	15	11.4	9.6	10.2	.55	.41	.45
	8-9	20	11.4	9.5	10.2	.56	.40	.45
	10-11	15	11.1	9.3	10.2	.55	.37	.43
	12-13	15	11.1	9.4	10.2	.54	.39	.44
	14	15	11.8	9.6	10.3	.57	.39	.46
	15	15	11.4	9.6	10.4	.55	.36	.45
	16	15	12.0	9.6	10.4	.56	.36	.45
	17	15	10.2	8.6	9.2	.62	.35	.49
Spruce-pine-fir	15	15	11.4	8.8	10.1	.39	.32	.35

Table 3.—Testing nail yield moments. Nail size, test variables, and resulting yield moment (mean and range of 10 replications)

Nail size	Length	Diameter	Span	Rate of loading	Yield moment mean (range)	Coefficient of variation
	------------------- *In* -------------------			*In/min*	*Lb in*	*Pct*
8d	2.5	0.131	1.77	0.12	40.4(37.7-42.6)	4.2
16d	3.5	0.162	2.76	0.16	77.1(70.3-81.3)	3.8
40d	5.0	0.225	3.94	0.25	186.7(179.2-198.9)	2.8
[1]40d	5.0	0.232	4.00	0.25	178.3(157.1-210.1)	9.4

[1]Nails used in test group 17 only.

Table 4.—Maximum embedding load (f_e) (mean of 15 replications)

Wood members	Wood thick-ness	Nail size[1]	Maximum embedding load f_e	Deformation at f_e	Coefficient of variation of f_e
		In.	*Lb/in.*	*In.*	*Pct*
Douglas-fir lumber	0.85	40d	1,422	0.15	18
	0.40	8d	788	0.13	22
Spruce-pine-fir lumber	0.85	40d	977	0.11	11
Plywood	0.45	8d	977	0.12	18
	0.47	40d	1,348	0.11	14

[1]40d nails were 0.225 in diameter, set into 0.220 in predrilled holes.
8d nails were 0.13 in diameter, set into 0.063 in predrilled holes.

The shape and dimensions of the specimens and nail patterns am given in figure 7 for each test group. All joints were assembled just before testing. The nails were driven in predrilled holes of 0.22-inch diameter for 40d nails and 0.063-inch diameter for 8d nails, the same as the embedding tests (table 4). The nailhead was always left 0.2-0.4 inch above the wood surface, except the test groups 7 and 8, where nailheads were driven flush to the surface. In most tests, 0.009-inch-thick shims were placed between the joint members during assembly and were removed shortly thereafter. For the three-member test specimens (tested in compression), the ends of the side members were adjusted so that they were level and perpendicular to the specimen's length direction.

The two-member specimens were attached to the testing machine by 0.45-inch bolts at both ends and tested in tension. Three-member joints were tested in compression. The arrangements for loading two- and three-member joints ate shown in figures 8 and 9.

For all the specimens, the load was applied at a constant deformation rate of 0.05 inch per minute. The three-member joints were tested in a 10,000-pound hydraulic testing machine. The joint deformation was assumed to be equivalent to the travel of the movable head of the machine. Two-member joints were tested in a 120,000-pound hydraulic testing machine. In this case a transducer measured the relative movement between the two members. In both tests, an x-y plotter continuously recorded load-deformation curves. Joint tests were stopped at "yield" defined by the Nordtest as maximum load or 0.22 to 0.3 inch, whichever occurred first.

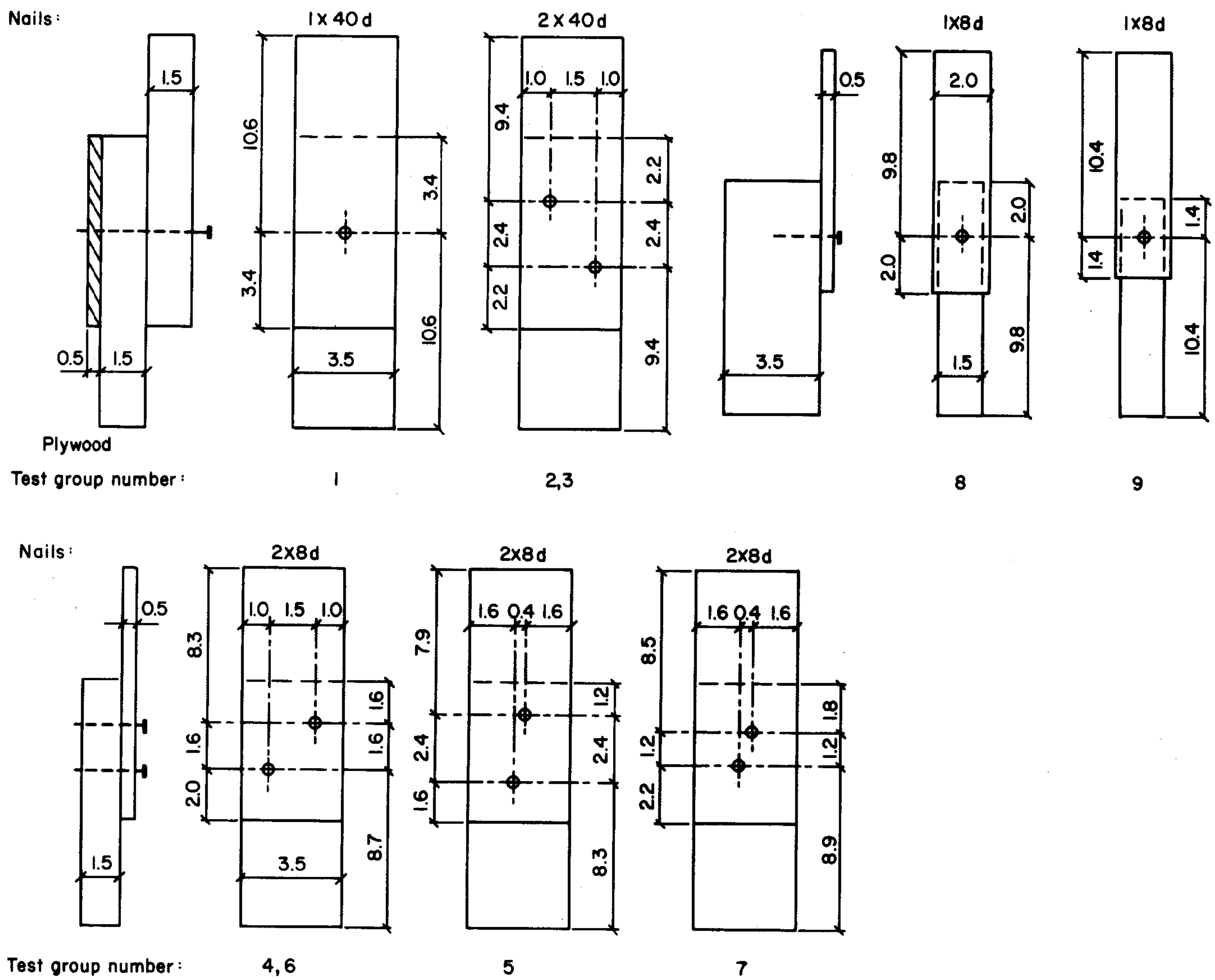

Figure 7.—Dimensions of nail test specimens and test group numbers. All dimensions are inches. (ML85 5344)

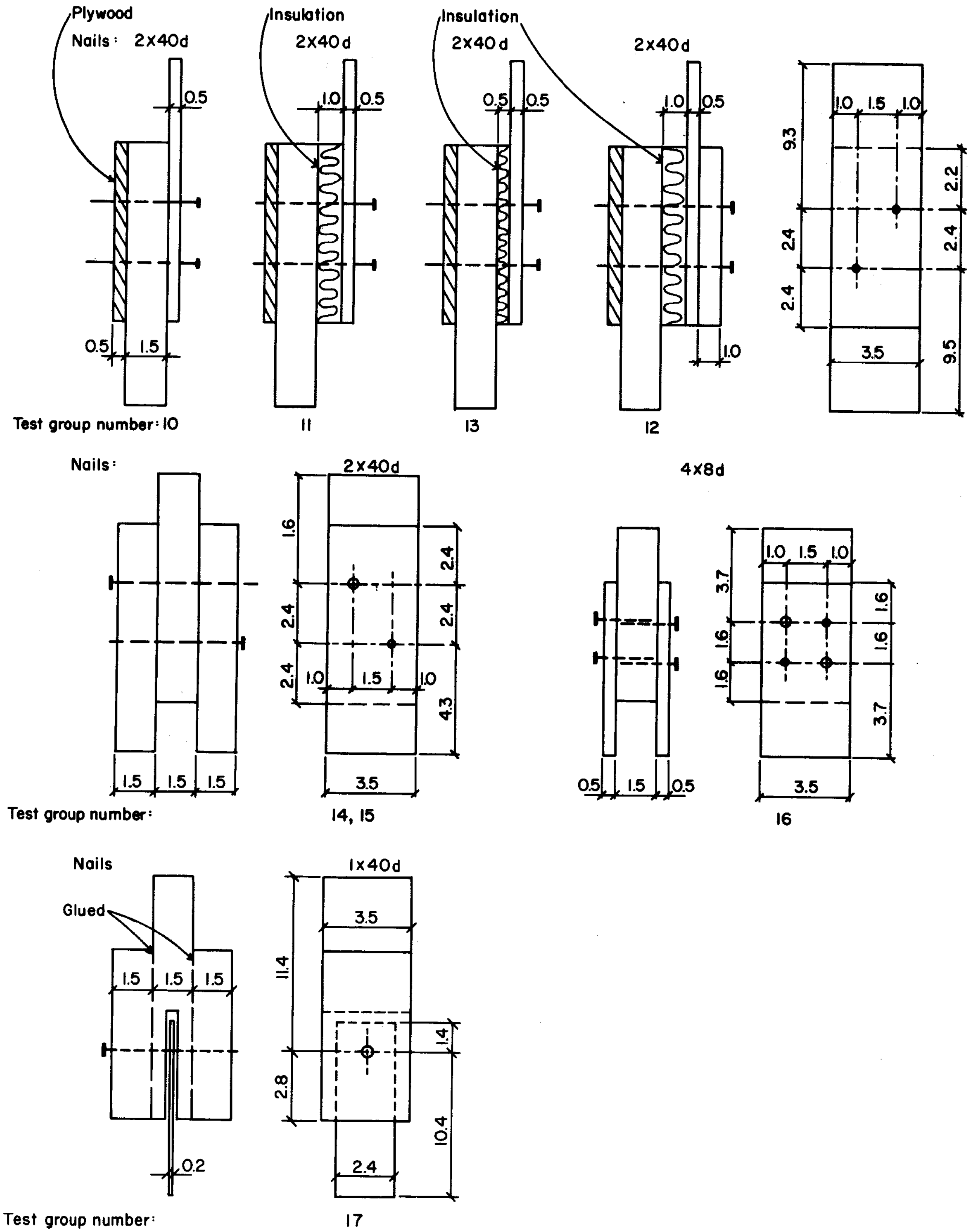

Figure 7.—Dimensions of nail test specimens and test group numbers. All dimensions are inches (continued). (ML85 5343)

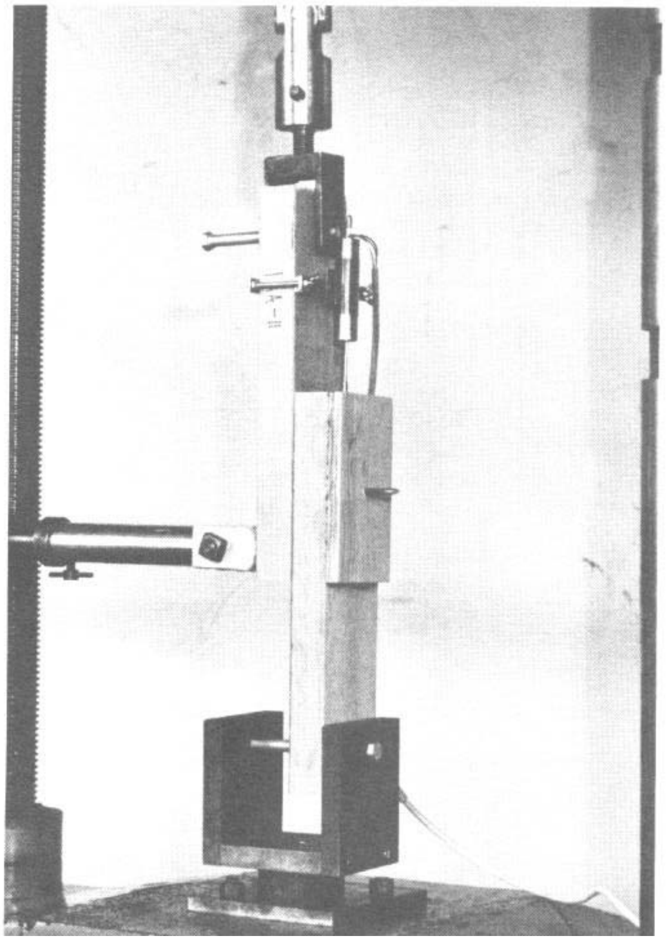

Figure 8.—Joint test apparatus, ASTM D 1761. loading a single-shear joint in tension. Nailheads were left 0.2-0.4 inch above the wood member in all but two test groups. Two test groups had nailheads driven flush in accordance with ASTM D 1761. (M 150804-10)

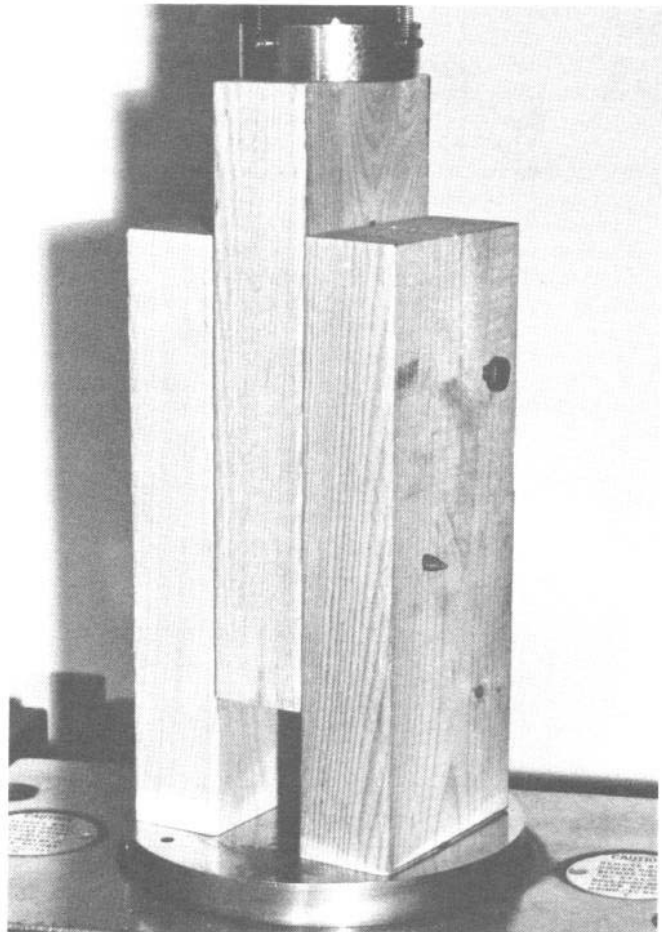

Figure 9.—Joint test apparatus, Nordtest, loading double-shear joint in compression. Nailheads were 0.2-0.4 inch above the wood member. (M 150804-3)

Results

Nail Yield Moments

Nails subjected to three-point bending resulted in average yield moments My ranging from 40.4 lb in. for 8d nails to 186.7 lb in. for 40d nails (table 3). Coefficients of variation (COV) were approximately 3-4 percent except for the second lot of 40d nails which had a COV of 9 percent. Similar investigations (Aune 1966) report COV in the range of 3 to 4 percent.

Assuming ideal yielding over the entire nail cross section, the yield moment M_y can be converted to yield strength f_y using the following formula:

$$f_y = \frac{M_y}{Z} = \frac{M_y}{d^3/6}$$

d = nail diameter (in.)
f_y = nail yield strength (lb/in.^{2})
M_y = nail yield moment (lb in.)
Z = plastic section modulus (in.^{3})

8d nail	f_y = 104,000 lb/in.^{2}
16d nail	f_y = 109,000 lb/in.^{2}
40d nail	f_y = 98,000 lb/in.^{2}

These calculations indicate nail strength depends on nail diameter. Smaller nails showed higher yield strength most likely because of strain hardening occurring during nail manufacture.

Nail yield angle can be derived from the curves of average moment versus angular rotation (shown in fig. 10). Yield occurred at an angle of approximately 0.05 radians for the three sizes of nails tested. In the actual nailed joint tests, yielding required a joint deformation of about 0.06 inch for 8d nails and of 0.10 inch for 40d nails. These deformations were obtained in all test groups.

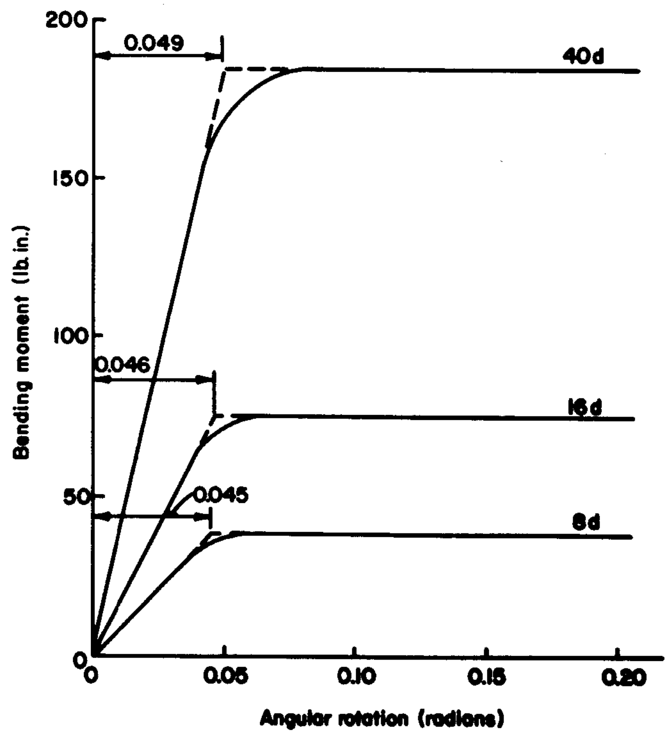

Figure 10.—Average nail moment plotted against angular rotation for 8d, 16d, and 40d nails. (ML85 5332)

Lumber Properties and Embedding Strength

The mean values of moisture content for all the test lumber ranged from 9 to 12 percent (table 2). Specific gravity ranged from 0.35 to 0.62 for Douglas-fir and from 0.32 to 0.39 for SPF (table 2). No corrections in the test results were made for variation in moisture content or specific gravity within a species group.

Plotting the mean load against deformation for the embedding stress tests produced the curves shown in figures 11a and 11b. Average embedding strengths (f_e) are given in table 4. Embedding strengths range from 977 lb/in. for SPF lumber with 40d nails to 1,422 lb/in. for Douglas-fir lumber with 40d nails. Coefficients of variation range from 11 to 22 percent.

Embedding strength is related to specific gravity in figure 12 for all Douglas-fir specimens with 40d and 8d nails. For 8d nails embedding strengths range from approximately 650 lb/in. at a specific gravity of 0.42 to 850 lb/in. at a specific gravity of 0.56. For 40d nails embedding strengths are 1,200 lb/in. and 1,750 lb/in., respectively, at these specific gravities.

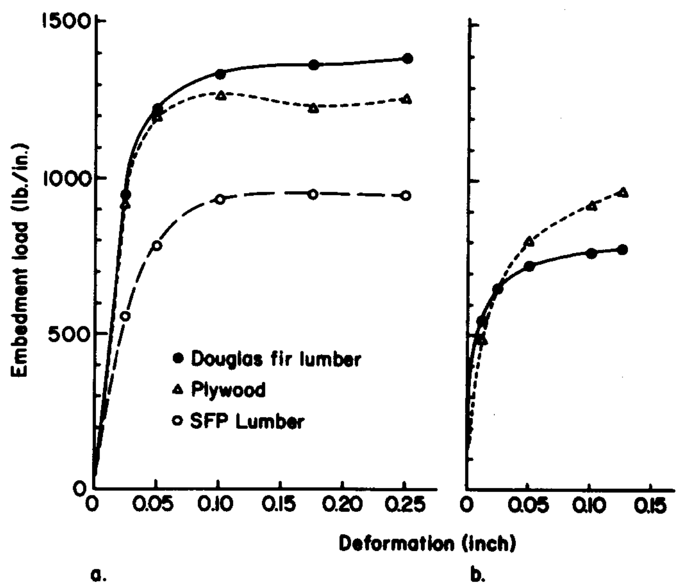

Figure I I .-Average embedding load plotted against deformation (a) for 40d nails with Douglas-fir, SPF lumber, and plywood, (b) for 8d nails with Douglas-fir lumber and plywood. (ML85 5345)

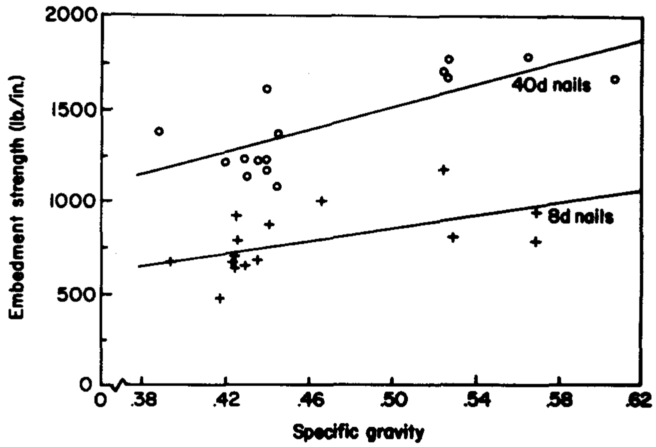

Figure 12.—Linear relationship between embedding strength and specific gravity indicated by regression lines for Douglas-fir lumber embedding 40d and 8d nails. (ML85 5333)

Nailed Joint Tests

The results of the nailed joint tests are shown in table 5. Average loads at selected deformations and the maximum load are given with their respective COV's. At all deflections the COV's ranged from 5 to 25 percent (a single COV at 36 pct). The highest readings are at the lower end of the load-deflection curve. Coefficients of variation on maximum load range between 4 and 15 percent.

Yield Theory Joint Strength

Failure modes and yield loads are predicted according to the formulas in the Appendix (tables A1-A8) by using the nail yield moment (M,), wood embedding strength (f_e), and specimen thickness. Predicted and observed modes of failure as well as predicted yield loads for all test groups are shown in table 6 together with ratios of the predicted yield loads to the test maximum loads (from table 5) for all test groups. Predicted loads were 79-115 percent of test maximum loads.

Table S.-Nailed joint test results: mean load per nail per shear plane at deformations δ with corresponding coefficients of variation (COV)

Test group No.	Loads and COV at deformation δ for—															
	δ = 0.1 inch		δ = 0.025 inch		δ = 0.05 inch		δ = 0.1 inch		δ = 0.15 inch		δ = 0.225 inch		δ = 0.3 inch		δ = Maximum	
	Load	COV	Load	COV	Load	COV	Load	COV	Load	COV	Load	COV	Load	COV	Load	COV
	Lb	*Pct*	*Lb*	*Pct*	*Lb*	*Pct*	*Lb*	*Pct*	*Lb*	*Pct*	*Lb*	*Pct*	*Lb*	*Pct*	*Lb*	*Pct*
1	183	22.4	323	22.1	466	21.2	616	16.4	689	13.8	739	12.3	767	11.9	767	11.9
2	184	21.1	309	19.7	431	18.8	568	14.7	640	12.1	692	10.1	724	9.6	724	9.6
3	225	13.3	344	14.6	462	15.2	594	13.3	653	11.4	692	9.9	708	9.2	708	9.2
4	72	26.3	115	15.6	149	10.3	178	8.4	193	8.9	[1]202	6.7			202	9.5
5	60	21.5	101	13.7	132	11.6	157	12.4	170	12.6	173	13.6			180	12.3
6	71	19.0	118	9.6	148	6.9	171	6.1	182	6.4	181	8.4			186	7.1
7	92	18.8	148	10.1	188	9.0	233	9.0	255	10.2	265	14.7			272	14.6
8	132	10.3	178	8.4	214	7.7	251	7.8	263	10.9	270	15.0			283	13.0
9	84	19.0	120	13.1	144	11.6	167	10.7	178	10.7	178	14.8			187	11.3
10	62	36.7	162	23.1	294	14.0	411	7.2	456	6.5	458	8.2			470	7.2
11	27	22.2	56	11.2	97	8.3	147	6.4	172	4.7	195	3.9	215	4.6	215	4.6
12	70	11.4	123	7.2	193	5.5	276	4.8	318	4.3	358	4.0	391	4.1	391	4.1
13	51	16.4	107	11.5	172	9.6	232	6.9	262	6.0	290	5.9	310	6.6	310	6.6
14	103	13.3	262	14.0	451	17.1	620	15.2	707	13.1	796	11.8	841	11.3	841	11.3
15	85	14.6	209	14.6	353	13.6	496	11.5	578	9.4	644	7.4	681	6.2	681	6.2
16	62	6.2	115	4.5	149	5.0	177	6.0	192	7.6	201	7.5	201	10.4	201	8.1
17	542	26.0	817	13.6	946	11.8	1,036	11.7	1,087	12.6	1,141	12.5	1,165	12.1	1,165	12.2

[1]Load at 0.24-in. deflection.

Table 6.—Modes of failure and yield loads (comparing predicted to observed values using average M_y and f_e values)

Test group number	Mode of failure number		Predicted yield load F_u	Ratio of predicted to test average
	Predicted	Observed		
			Lb	
[1]1	1.4	1.4 (75 pct) 1.3 (25 pct)	728	0.95
2	3.4B	3.4B	704	0.97
3	3.4B	3.4B	697	0.98
4	1.3	1.3	216	1.07
5	3.3B	3.3B	202	1.12
6	3.3B	3.3B	200	1.08
[2]7	– –	– –	– –	– –
[2]8	– –	– –	– –	– –
9	1.3	1.3	216	1.15
10	1.3	1.3	513	1.09
11	3.3B	3.3B	200	0.93
12	3.4B	3.4B	307	0.79
13	3.3B	3.3B	315	1.01
[1]14	2.4	2.4 (75 pct) 2.3 (25 pct)	728	0.87
15	2.3	2.3	643	0.94
16	2.3	2.3	216	1.07
17	2.2SB	2.2SB	1,030	0.88

[1]Maximum M_y and minimum f_e values used in prediction.

[2]Yield theory was not used to predict maximum loads for joints with nailheads driven flush to the surface.

Discussion

The yield theory provides a method to analyze nailed joints using a material science approach. The two material properties required for the analysis are wood embedding strength and nail yield moment.

Embedding Strength

The Nordtest (1981) specifies the wood embedding strength is to be determined from a test which embeds the nail at least 0.1 inch up to a depth of one nail diameter. With plywood, however, the maximum load occurred at embedments greater than one nail diameter in a majority of the tests, Plywood load-deflection curves showed continuously increasing embedding stress with increasing deflection. In contrast, mote than two-thirds of the solid wood specimens reached a maximum load by embedment of one nail diameter. Embedding stress in solid wood was also nearly constant above a certain deflection. Therefore, the assumption of plasticity seems appropriate for solid wood, but is less assured for plywood.

Figure 12 is a plot of the individual embedding strength versus specific gravity for Douglas-fir test groups. The plot shows embedding strength to be roughly proportional to specific gravity. (Regression equations for Douglas-fir lumber are: f_e = 3,28l(SG) - 117 for 40d nails and f_e = l,719(SG) - 9 for 8d nails where SG = specific gravity.) other investigations (Ehlbeck 1979) note similar trends in embedding strength data.

Nail Joint Tests

Predicted Yield Load Versus Test Results

Wood embedding strength was more variable than nail yield moment (tables 3 and 4). Therefore, failure modes which depend on nail yield had less variable ultimate loads than failure modes more dependent on wood embedment.

For example, the three test groups involving insulation (groups 11, 12, and 13) showed the lowest COV on ultimate load (table 5) primarily because the ultimate load is a function of nail bending rather than wood embedment. Also, the highest COV's in the test of the joints (table 5) were recorded at the lower end of the load-deformation curve where wood embedment is an important factor.

The sensitivity of predicted yield load (F_u) to changes in nail yield (M_y) and wood embedment (f_e) are described below for joints with an intermediate layer of insulation, failure mode 3.4B (table A7, Appendix).

Increasing f_e by 10 percent increases F_u by 1.5 percent.

Increasing f_e by 20 percent increases F_u by 2.8 percent.

Increasing M_y by 10 percent increases F_u by 8.3 percent.

Increasing M_y by 20 percent increases F_u by 16.4 percent.

The effect of having a lower embedding strength in the side member can be seen in the joint with SPF side member (test group 15, table 6). The predicted yield load is 87 percent of the predicted yield load of a joint where both members ate Douglas-fir (test group 14, table 6). For group 15 the load-deformation curve (at any deformation) is 80-83 percent of the comparable group 14 test (fig. 13a).

The predicted yield loads and the modes of failure based on the mean values of M_y and f_e coincide fully with observed modes for all test groups except Douglas-fir lumber with 40d nails (test groups 1 and 14, table 6). Using maximum M_y and minimum f_e values made possible the occurrence of either one of two failure modes for these test groups. Predicted yield loads are within 20 percent of average test values.

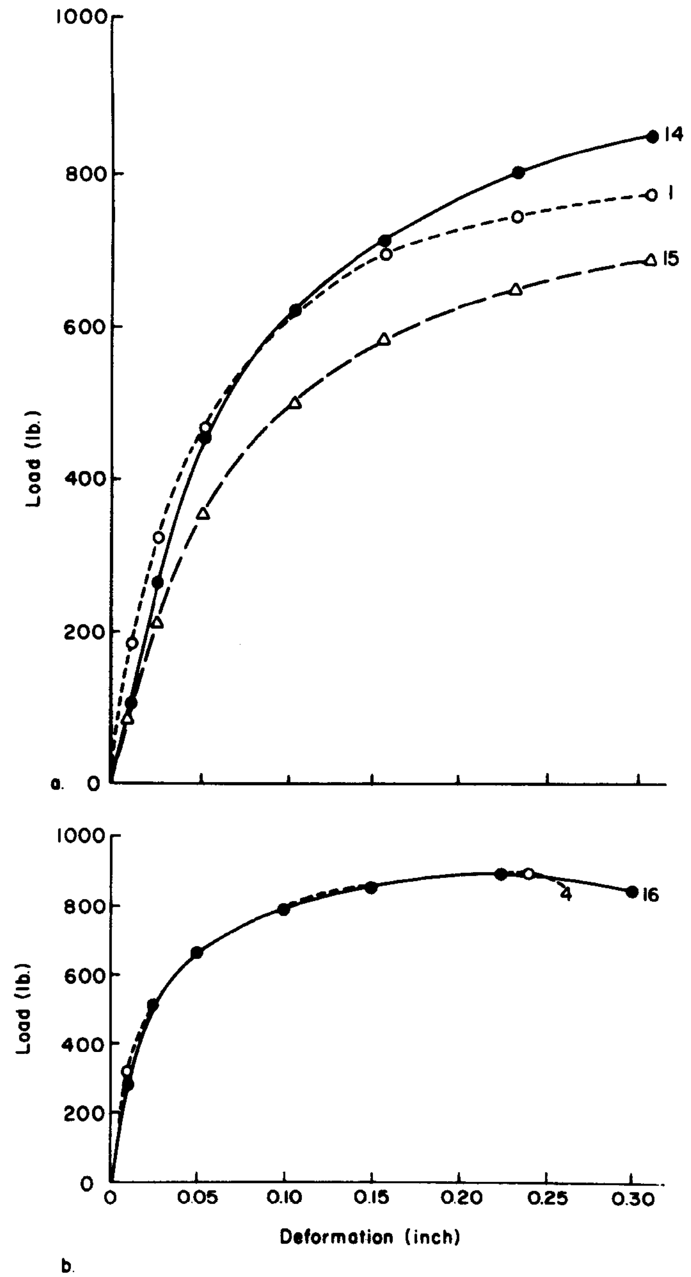

Figure 13.—(a) Two-member joints (test group I) compare favorably with three-member joints (test group 14) using 40d nails with nailheads 0.2-0.4 inch above the member surface. Embedding strength in the side member: compare joints with SPF side members (test group 15) and Douglas-fir side members (test group 14).
(b) Two-member joints (test group 4) compare favorably with three-member joints (test group 16) using 8d nails with plywood side members and nailheads 0.2-0.4 inch above the surface. (ML85 5346)

Single- and Double-Shear Joints

Comparison is made in figure 13a between the two-member test setup (test group 1) and the three-member test setup (test group 14) for 40d nails (Douglas-fir). The two-member test shows a higher load at low deformations. The two load-deformation curves **are** almost equal at 0.07-o. 12 inch deformation. Beyond that point the three-member test shows a continuously increasing load compared to the two-member test. The difference, however, is less than 10 percent.

Comparison is made in figure 13b between two-member (test group 4) and three-member (test group 16) tests for 8d nails, using Douglas-fir lumber and plywood. At the O.Ol-inch deformation, the two-member test shows about 15 percent higher load than the three-member test. From the **0.03-inch** deformation point, the two loaddeflection curves coincide completely (a load difference less than 0.5 pct). In both cases, we conclude that there is no significant difference in the results from the two- and the three-member test setup.

Friction and Gaps

The yield model does not include the effects of friction between members nor axial forces in the nail. The presence of these two forces increases the lateral load capacity of a nailed joint. Friction is ignored because nailed joints in a structure may not have nailheads flush to the surface (allowing axial forces to develop in the nail) and may not be in tight contact after exposure to shrinkage cycles in the wood.

The ASTM standard D 1761 (1977) specifies that nailheads should be **driven** flush to the member's surface, putting the members in tight contact. In the Nordtest (1981) nailheads are left above the surface which results in members not being held in tight contact. The Nordtest is a three-member joint test, whereas the ASTM test is a two-member joint test.

Test specimens listed in table 1 include joints with and without friction, with varying size of gaps and position of nailheads, for both single-shear and double-shear joint tests. However, similar load-deformation behavior and ultimate loads resulted when either a single-shear or double-shear test was assembled with nailheads not driven flush (figs. 13a and 13b).

For deflection readings between 0.01 to 0.225 inch, test group 7 has 26-32 percent higher mean loads than test group 4. This can be called the "nailhead" effect which may not exist in wood joints after shrinkage cycles have occurred.

Compare the load-deformation curves (fig. 14) for test group 4 which has the nailhead 0.2-0.4 inch above the surface, and test group 7 which has the nailhead driven flush against the member as required by ASTM D 1761. In both groups 0.009-inch shims are used during assembly to reduce friction. For both cases the gap would theoretically close at the same deformation.

During testing, the side member in test group 4 tends to slide away from the main member. There is neither friction nor a contribution from the nail's axial force. In test group 7, however, the nail's axial force develops because of the nailhead fixity. This force gradually contributes to the lateral load bearing of the joint.

The effect of having both friction and nailhead fixity is shown in figure 15. Compare load-deformation curves for the test group made entirely according to ASTM D 1761 (test group 8), having nailheads flush with the surface and no shims, to the test group having 0.009-inch shims and nailheads 0.2-0.4 inch above the surface (test group 9). The effect of nailhead fixity and friction, which is present in the ASTM test (test group 8), gives a load increase of 47-56 percent for all deformations compared to the Nordtest method (test group 9).

Effects of gap sizes ranging from 0.009 to 0.040 inch are small (less than 10 pct) in joints without nailheads driven flush to the surface (the Nordtest) when compared to the "nailhead" and "friction" effects mentioned above (see fig. 16). When the nailhead is driven flush and side members are relatively thick, an initial gap tends to close during loading, introducing friction. Relatively thin side members, represented by the plywood tests, did not develop this friction as the test progressed because the nail was withdrawn from the main member.

The ASTM test method requires nailheads to be driven flush to the surface. Research (Antonides et al. 1980) indicates that gaps have been an important variable when nailheads were driven flush to the surface in tests whose purpose was to evaluate joint stiffness. In contrast, the gap effect is insignificant in the present study when nailheads were not driven flush to the surface. We conclude that the most reproducible test method is like the Nordtest, one in which nailheads are not driven flush to the surface, and the hard to control "gap effect" does not arise.

The yield load of joints having a maintained gap (test groups 3 and 6) can be calculated using the equations derived from the yield theory applied to joints with an intermediate layer of insulation (table A7, Appendix). The predicted loads are 98-108 percent of the average test loads for test groups 3 and 6, respectively (table 6). Both joints had a 0.040-inch gap maintained throughout the test, and nailheads left 0.2-0.4 inch above the surface (Nordtest).

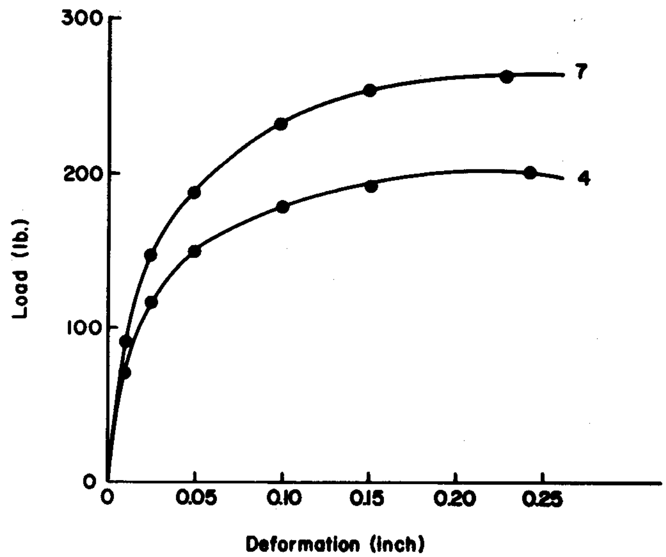

Figure 14.—The effect of nailhead fixity: test group 7 had 8d nailheads driven flush, whereas test group 4 had nailheads left 0.2-0.4 inch above the surface. Both test groups had 0.009-inch gap between members. (ML85 5334)

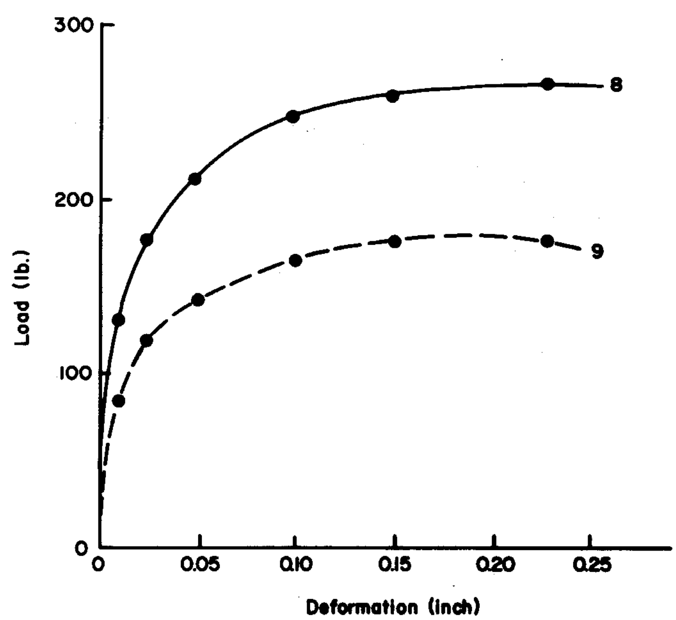

Figure 15.—The effect of both nailhead fixity and friction: test group 8, made according to ASTM D 1761, had nailheads driven flush and no shims, whereas test group 9 had nailheads 0.2-0.4 inch above the surface and 0.009-inch gap between members. (ML85 5335)

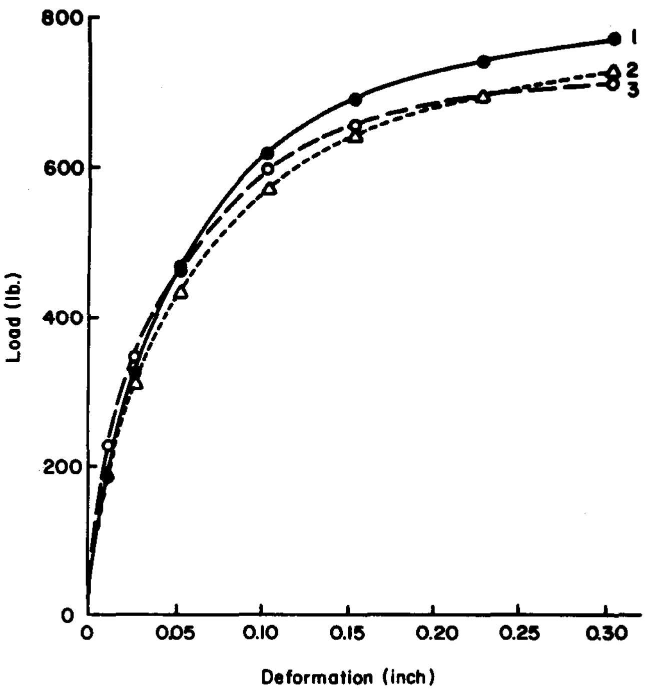

Figure 16.—The effect of gaps is small, for gap widths 0.009 inch (test group 1) to 0.036 inch (test group 2) and 0.040 inch (test group 3). when nailheads are left 0.2-0.4 inch above the surface. (ML85 5336)

Steel Plate Members

Joints with a steel member have minimal contribution from friction and thus their behavior is modeled well using the yield theory.

The basic theory (considering failure modes 2.4 and 2.2SB) predicts that replacing the wooden central member by a steel plate will increase yield load by 40 percent (tables A3 and A4, Appendix). The tests showed an increase of 38-52 percent in ultimate load (table 7), thus confirming the theory.

Steel plate members increase stiffness of a joint. In figure 17 we compare the test groups with all-wooden members in single and double shear (test groups 1 and 14) to the test group with a steel center member in double shear (test group 17). Especially in the low deformation range, the joint with a steel member has a much higher load than either of the wood-to-wood joints, up to 400 percent (table 7).

At low deformations nail bending contributes more to the load bearing of joints with a steel member than of joints with all-wood members. Also, as the wood-to-steel joint deforms the wood embedding stress increases with wood embedment (deformation). The wood deformation due to wood embedment in the case of a steel-to-wood joint is nearly twice the wood deformation seen in each member of a wood-to-wood joint.

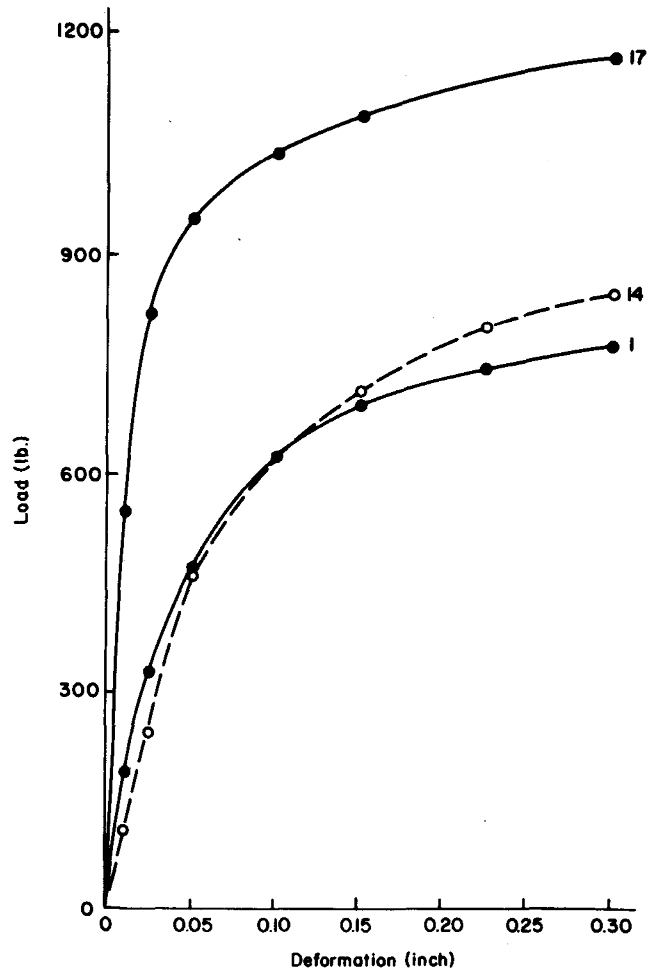

Figure 17.—Test group 17 with a steel member is considerably stiffer and has a higher ultimate load than comparable test groups 1 and 14 with all-wooden members. (ML85 5337)

Insulation Layer

The introduction of a layer of insulation is a natural extension from having a gap in the joint. Yield loads predicted from theory were 7-12 percent higher than those obtained by tests (table 6). In figure 18 we compare test groups 11 and 13, which have a layer of insulation, to the similar test group 10 which has no insulation. The 0.5-inch and 1.0-inch layer of insulation decrease both the ultimate load and stiffness of joints.

A wooden block under the nailhead (test group 12) induced an additional yield point in the nail, theoretically increasing the yield load about 50 percent (compared to test group 11, table 6). Test showed an even higher increase in ultimate load: 80-90 percent (table 5). The theory does not include the axial force developed in the nail which contributes to the test joint's lateral load capacity. The wooden block also increased the joint stiffness (fig. 18).

Fourth-Root Curve

The yield load can be expressed as a function of the joint deformation by assuming the relationship between wood embedding stress and deformation to be defined by a fourth-root curve. The general shape of this relationship compares well with data from embedment tests above 0.15-inch deformation (fig. 19).

We derived the fourth-root relationship of embedding stress to deformation from the combined steel-to-wood test group 17 and used it to predict the load-deformation curve of single-shear Douglas-fir joints (test group 1) in figure 19. For steel-to-wood test group 1, wood embedding is approximately equal to joint deformation; thus a special test method is not necessary to determine wood embedding behavior and we derived wood embedment behavior from a formula fitted to the joint load-deformation curve (see Appendix B, where we propose a method to derive the fourth-root embedding relationship from the combined steel-to-wood test). We found good agreement between the formula and test values for deformations above 0.1 inch. An adjustment of angular rotation for true moment of the nail would lower the joint deformation between 0.0 and 0.8 inch.

This method may represent an alternative way of computing a nail joint's yield load and load-deformation curve in the higher deformation range. For the lower part of the curve, adjustments are necessary to reflect the true moment-deflection relationship of the nail.

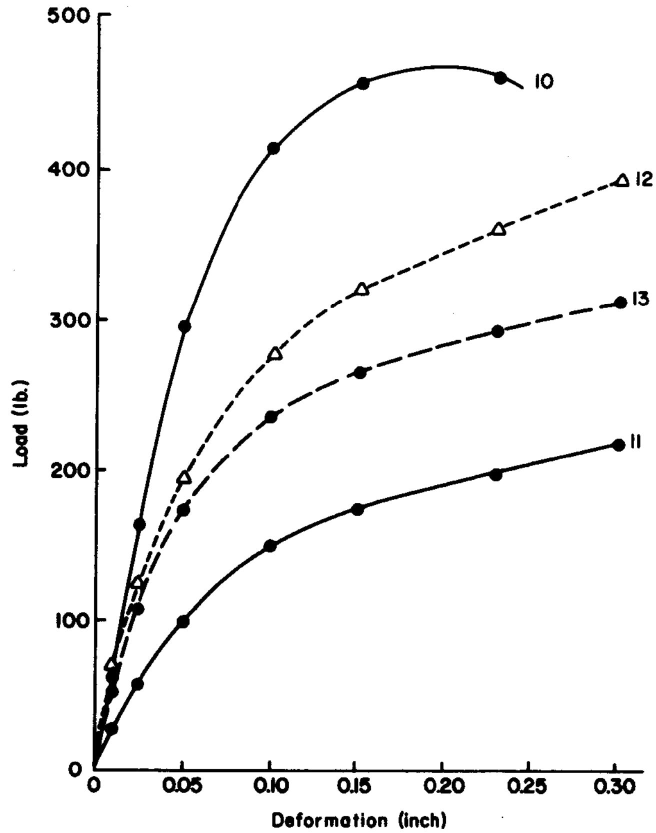

Figure 18.—Test groups 11 and 13, which had a layer of insulation, had lower stiffness and lower ultimate load than test group 10 which lacked insulation. A wooden block under the nailhead of joints with insulation (test group 12) increased both the stiffness and ultimate load. (ML85 5338)

Conclusions

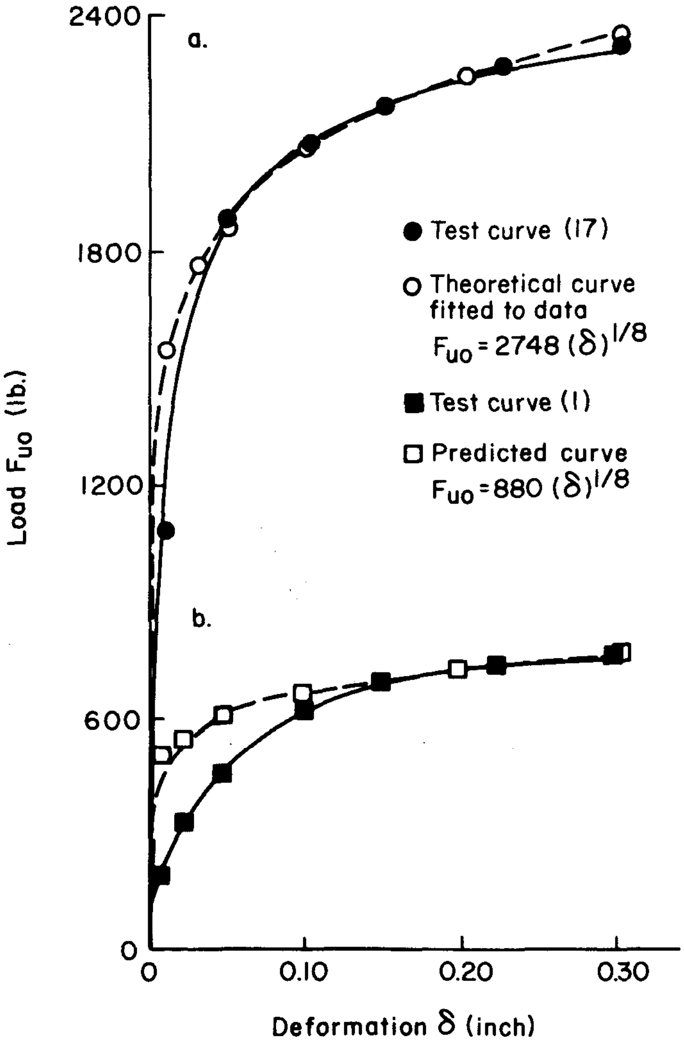

Figure 19.—(a) The steel-to-wood (test group 17) load-deformation curve used to derive the embedding strength fourth-root curve (test group 1). (b) Theoretical curve (derived for test group 17) predicting the load-deformation behavior of the woo&to-wood joint (test group I) compared with observed test curve. (ML85 5339)

Two- and three-member joint tests were conducted to verify the yield theory for nailed joints typical of construction practices in the United States. Formulas using wood embedding strength and nail yield moments predicted yield loads which were compared to test data. Finally, the possibility of predicting joint deformations using a modified yield theory was investigated.

From the results we concluded:

1. The yield theory accurately predicts yield load of nailed joints, including effects of different embedding strengths, different joint geometries, steel center plates, and gaps made by intermediate layers of insulation.

2. The combined effect of joint members being in tight contact (eliminating gaps) and driving nailheads flush to the surface, as in the ASTM test, is to increase lateral load throughout the load-deformation curve by about 50 percent. Since neither of these effects may exist in a real joint, the ASTM method is not a conservative estimate of the strength of joints in service.

3. Comparing specimens with an equal initial gap between members, the effect of driving nailheads flush to the surface is to increase the lateral load by 30 percent.

4. When nailheads are left above the surface and small gaps (0.04 in.) exist between members, there is no significant difference between results for three-member compression tests and two-member tension tests, either in load-deformation curves or ultimate loads.

5. The fourth-root wood embedding stress function may be useful for computing the load-deformation curve of a nailed joint using a modified yield theory. However, more investigations are needed.

Table 7.—Ratios between test loads at particular deformations for double-shear joint with a steel center member (teat group 17) and joints with all-wood members (test group 1, single-shear, and test group 14, double-shear). Load ratios are load per nail per shear plane

Test group ratio	Ratio of loads at deformations δ (in.) for–						
	δ=0.01	δ=0.025	δ=0.05	δ=0.10	δ=0.15	δ=0.225	Ultimate load
17/1	2.96	2.53	2.03	1.68	1.58	1.54	1.52
17/14	5.25	3.12	2.10	1.67	1.54	1.43	1.38

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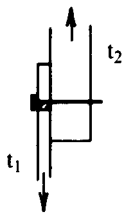
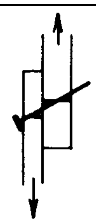
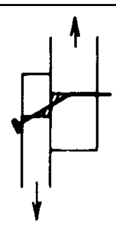
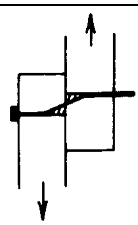
Appendix A
Formulas for Nailed Joints Based on the Yield Theory

The following tables are reproduced from Aune and Patton-Mallory (1986). They provide formulas for yield load for wood-to-wood and steel-to-wood joints including joints with dissimilar embedment strength of wood members. Additional formulas apply when a layer of insulation (or gap) exists between joint members. A final formula represents the relationship between the wood's embedding stress and deformation as a fourth-root curve.

Table Al.-Formulas and failure modes for two-member joints with both members of the same wood embedding strength. Member thickness conditions determine the failure mode

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
1.1		0	$F_u = f_e \cdot t_1$	$\frac{t_1}{\sqrt{\gamma}} \le 1$ $\alpha \ge 3$
1.2		0	$F_u = \frac{f_e \cdot t_1}{2}\left[\sqrt{3\alpha^2 + 2\alpha + 3} - (1 + \alpha)\right]$	$\frac{t_1}{\sqrt{\gamma}} < \frac{4}{(1 + 3\alpha) - \sqrt{3\alpha^2 + 2\alpha + 3}}$ $1 < \alpha < 3$
1.3		1	$F_u = \frac{f_e}{3}\left[2\sqrt{t_1^2 + 3\gamma} - t_1\right]$	$1 < \frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{2}$ $t_2 \ge \frac{2}{3}\sqrt{t_1^2 + 3\gamma} - \frac{1}{3}t_1 + 2\sqrt{\gamma}$
1.4		2	$F_u = \sqrt{2f_e \cdot M_y}$	$\frac{t_1}{\sqrt{\gamma}}$ and $\frac{t_2}{\sqrt{\gamma}} \ge 2 + \sqrt{2}$

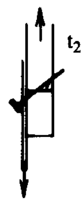
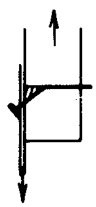
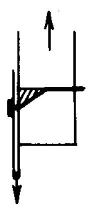
f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\alpha = t_2/t_1$

$\gamma = M_y/f_e$

Table A2.—Formulas and failure modes for two-member joints with one steel member. Member thickness conditions determine the failure mode

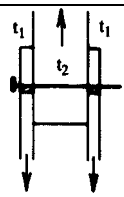
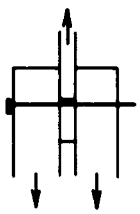
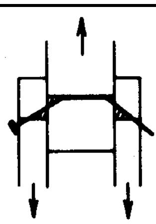
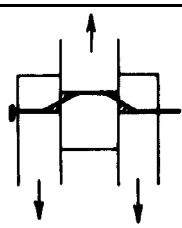
Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
1.2S		0	$F_u = f_e \cdot t_2 (\sqrt{2} - 1)$	$\frac{t_2}{\sqrt{\gamma}} < 2 + \sqrt{2}$
1.3S		1	$F_u = \sqrt{2f_e \cdot M_y}$	$\frac{t_2}{\sqrt{\gamma}} \geq 2 + \sqrt{2}$
1.4S		2	$F_u = \sqrt{2}\sqrt{2f_e \cdot M_y}$	$\frac{t_2}{\sqrt{\gamma}} > 4$

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\gamma = M_y/f_e$

Table A3.—Formulas and failure modes for three-member joints with all members of same wood embedding strength. Member thickness conditions determine the failure mode

Mode of failure number	**Failure** geometry	**Number of nail yield points**	**Yield load F_u (lb)**	**Thickness conditions**
2.1		0	$F_u = 2f_e \cdot t_1$	$0 < \frac{t_1}{\sqrt{\gamma}} \leq 1$ $\alpha > 2$
2.2		0	$F_u = f_e t_2 = f_e \cdot \alpha t_1$	$\alpha < 2$ $\frac{t_2}{\sqrt{\gamma}} < 2\sqrt{2}$
2.3		2	$F_u = \frac{2}{3} f_e \left[2\sqrt{t_1^2 + 3\gamma} - t_1 \right]$	$1 < \frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{2}$ $t_2 \geq \frac{2}{3} \left[2\sqrt{t_1^2 + 3\gamma} - t_1 \right]$
2.4		4	$F_u = 2\sqrt{2f_e \cdot M_y}$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + \sqrt{2}$ $\frac{t_2}{\sqrt{\gamma}} \geq 2\sqrt{2}$

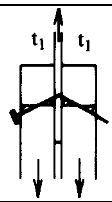
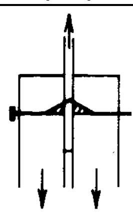
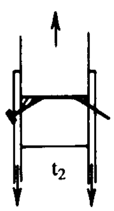
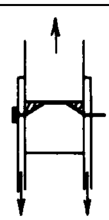
f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

$\alpha = t_2/t_1$

$\gamma = M_y/f_e$

Table A4.—Formulas and failure modes for three-member joints with a steel center member, or steel side members. Member thickness conditions determine the failure mode. All wood members have the same embedding strengths

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
2.2SA		[1]2	$F_u = 2f_e\left[\sqrt{2}\,\sqrt{t_1^2 + 2\gamma} - t_1\right]$	$\sqrt{2} < \frac{t_1}{\sqrt{\gamma}} < 4$
2.2SB		[1]4	$F_u = 4\sqrt{f_e M_y}$	$\frac{t_1}{\sqrt{\gamma}} \geq 4$
2.3S		2	$F_u = 2\sqrt{2 f_e M_y}$	$\frac{t_2}{\sqrt{\gamma}} \geq 2\sqrt{2}$
2.4S		4	$F_u = \sqrt{4 f_e M_y}$	$\frac{t_2}{\sqrt{\gamma}} \geq 4$

[1]The middle yield point is regarded as two yield points in the ultimate load formula.

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

γ = M_y/f_e

Table A5.—Formulas and failure modes for two-member wood joints. Wood members (t_1 and t_2) have unequal embedding strengths (f_e and βf_e lb/in. respectively). Member thickness conditions determine the failure mode

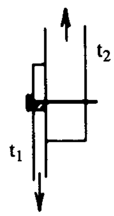
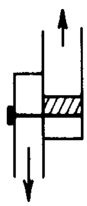
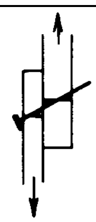
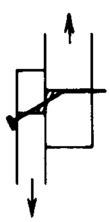
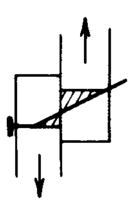
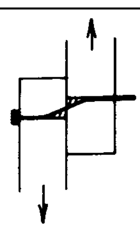
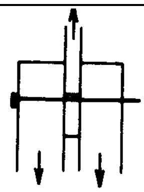
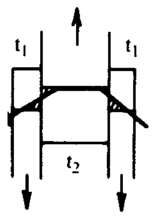
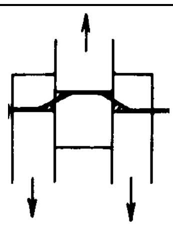
Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
1.1		0	$F_u = f_e \cdot t_1$	$\frac{t_1}{\sqrt{\gamma}} < \sqrt{\frac{2\beta}{1+\beta}}$ $t_2 > \frac{t_1}{\beta}$
1.1A		0	$F_u = \beta f_e t_2$	$\frac{t_2}{\sqrt{\gamma}} < \sqrt{\frac{2}{\beta(1+\beta)}}$ $t_1 > \beta t_2$
1.2		0	$F_u = f_e \cdot t_1 \left[\frac{\sqrt{\beta + 2\beta^2(1+\alpha+\alpha^2) + \alpha^2\beta^3} - \beta(1+\alpha)}{1+\beta}\right]$	$\frac{t_1}{\sqrt{\gamma}} < \frac{(1+\beta)}{(2\beta + 1 + \alpha\beta) - \sqrt{\beta + 2\beta^2(1+\alpha+\alpha^2) + \alpha^2\beta^3}}$ $\frac{1}{\beta} < \alpha < \frac{1 + \sqrt{2(1+\beta)}}{\beta}$
1.3		1	$F_u = f_e \left[\sqrt{\frac{2\beta(1+\beta)t_1^2}{(2+\beta)^2} + \frac{4\beta\gamma}{(2+\beta)}} - \frac{\beta t_1}{(2+\beta)}\right]$	$\frac{t_1}{\sqrt{\gamma}} > \sqrt{\frac{2\beta}{1+\beta}}$ and $2 - 2\sqrt{\frac{\beta}{1+\beta}} < \frac{t_1}{\sqrt{\gamma}} < 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $t_2 \geq \sqrt{\frac{2(1+\beta)t_1^2}{\beta(2+\beta)^2} + \frac{4\gamma}{\beta^2(2+\beta)}} - \frac{t_1}{(2+\beta)} + \frac{2\sqrt{\gamma}}{\sqrt{\beta}}$
1.3A		1	$F_u = f_e \cdot \beta \left[\sqrt{\frac{2\alpha^2(1+\beta)t_1^2}{(1+2\beta)^2} + \frac{4\gamma}{\beta(1+2\beta)}} - \frac{\alpha t}{(1+2\beta)}\right]$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $\frac{t_2}{\sqrt{\gamma}} = \frac{\alpha t_1}{\sqrt{\gamma}} > \sqrt{\frac{2}{\beta(1+\beta)}}$ and $\frac{1}{\sqrt{\beta}}\left[2 - 2\sqrt{\frac{1}{(1+\beta)}}\right] < \frac{\alpha t_1}{\sqrt{\gamma}} < \frac{1}{\sqrt{\beta}}\left[2 + 2\sqrt{\frac{1}{(1+\beta)}}\right]$
1.4		2	$F_u = \sqrt{\frac{\beta \cdot 4f_e \cdot M_y}{(1+\beta)}}$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $\frac{t_2}{\sqrt{\gamma}} \geq \frac{1}{\sqrt{\beta}}\left[2 + 2\sqrt{\frac{1}{(1+\beta)}}\right]$

Table A6.—Formulas and failure modes for three-member wood joints. Wood members (t_1 and t_2) have unequal embedding strengths (f_e and βf_e lb/in respectively). Thickness conditions determine the failure mode

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Thickness conditions
2.2		0	$F_u = \beta f_e t_2 = \beta f_e \alpha t_1$	$t_1 > \beta t_2$ $\frac{t_2}{\sqrt{\gamma}} < \frac{4}{\sqrt{\beta(1+\beta)}}$
2.3		2	$F_u = 2f_e\left[\sqrt{\frac{2\beta(1+\beta)t_1^2}{(2+\beta)^2} + \frac{4\beta\gamma}{(2+\beta)}} - \frac{\beta t_1}{(2+\beta)}\right]$	$\frac{t_1}{\sqrt{\gamma}} < 2 + \sqrt{\frac{\beta}{(1+\beta)}}$ $t_2 \geq \frac{2\sqrt{2\beta(1+\beta)t_1^2 + 4\beta(2+\beta)\gamma} - 2\beta t_1}{\beta(2+\beta)}$
2.4		4	$F_u = 2\sqrt{\frac{\beta 4 f_e \cdot M_y}{(1+\beta)}}$	$\frac{t_1}{\sqrt{\gamma}} \geq 2 + 2\sqrt{\frac{\beta}{1+\beta}}$ $\frac{t_2}{\sqrt{\gamma}} \geq \frac{4}{\sqrt{\beta(1+\beta)}}$

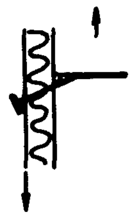
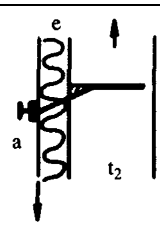
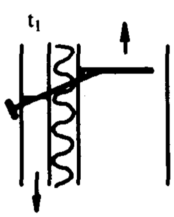
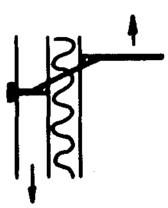
f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

α = t_2/t_1

γ = M_y/f_e

Table A7.—Formulas and failure modes for two-member sheathing to wood joints with an intermediate layer of insulation (e). Wood has embedding strength f_e. Thickness conditions and theoretical nail length determine the failure mode

Mode of failure number	Failure geometry	Number of nail yield points	Yield load F_u (lb)	Failure mode determinants
3.3A		1	$F_u = f_e\sqrt{e^2 + 2\gamma} - e$	Theoretical length of nail: $\ell = \sqrt{e^2 + 2\gamma} + 2\sqrt{\gamma}$
3.4A		2	$F_u = f_e\left[\sqrt{e^2 + 4\gamma} - e\right]$	Theoretical length of nail: $\ell = \sqrt{e^2 + 4\gamma} + 2\sqrt{\gamma} + a$
3.3B		1	$F_u = \frac{f_e}{3}\left[2\sqrt{t_1^2 + et_1 + e^2 + 3\gamma} - (t_1 + 2e)\right]$	Thickness conditions: $\frac{\sqrt{e^2 + 4\gamma} - e}{2} < t_1 < \frac{\sqrt{e^2 + 8\gamma} - e + 4\sqrt{\gamma}}{2}$ Theoretical length of nail: $\ell = \frac{2t_1 + e}{3} + \frac{2}{3}\sqrt{t_1^2 + et_1 + e^2 + 3\gamma} + 2\sqrt{\gamma}$
3.4B		2	$F_u = \frac{f_e}{2}\left[\sqrt{e^2 + 8\gamma} - e\right]$	Thickness conditions: $t_1 \geq \frac{\sqrt{e^2 + 8\gamma} - e + 4\sqrt{\gamma}}{2}$ Theoretical length of nail: $\ell = \sqrt{e^2 + 8\gamma} + 4\sqrt{\gamma}$

a = thickness of cleat under nailhead (in)

e = thickness of intermediate layer of insulation (in)

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

γ = M_y/f_e

Table A8.—Formulas and failure modes for three-member joints with steel and wood members, comparing estimates of yield loads assuming a plastic relationship (F_u) or assuming a fourth-root curve (F_{uo}) between wood embedding stress and deformation

Mode of failure number	Number of nail yield points	Estimates of yield load		Ratio F_{uo}/F_u
		F_{uo}	F_u (table A4)	
2.2SB	4	$\frac{8\sqrt{2f_1M_y}\cdot(\delta)^{1/8}}{3(\eta_1)^{1/8}}$	$4\sqrt{f_eM_y}$	0.94
2.4	4	$\frac{8\sqrt{f_1M_y}\cdot(\delta)^{1/8}}{3(\eta_1)^{1/8}}$	$2\sqrt{2f_eM_y}$	0.94

f_1 = wood embedding stress at deformation η_1

f_e = wood embedding strength (lb/in)

M_y = nail yield moment (lb in)

F_u = yield load of joint assuming plastic wood embedment (lb)

F_{uo} = yield load of joint assuming fourth-root wood embedment (lb)

δ = joint deformation (in)

η = wood embedding deformation (in)

η_1 = wood embedding deformation corresponding to embedding stress f_1 (in)

Appendix B
Derivation of Fourth-Root Wood Embedment From Steel-to-Wood Load-Deformation Curve

It is of interest to express a joint's ultimate load as a function of joint deformation. Assume the combined steel-to-wood joint (test group 17) provides the basic information about wood embedding strength for Douglas-fir. Also, assume the embedding strength takes the form of a fourth-root curve, resulting in the following formula (table A8, Appendix A) for the ultimate load of the joint.

$$F_{uo} = \frac{8}{3}\cdot\sqrt{2f_1M_y}\cdot\left(\frac{\delta}{\eta_1}\right)^{1/8}\cdot\frac{1}{2}$$

f_1 = wood embedding stress at deformation η_1 (lb/in.)
F_{uo} = ultimate joint load (lb)
M_y = nail yield moment (lb in.)
δ = joint deformation (in.)
η_1 = wood embedding deformation (in.)

The factor 1/2 accounts for 2 shear planes in test group 17. At deformation $\eta_1 = \delta$

$$f_1 = \frac{9F_u^2}{16\cdot 2\,M_y}$$

Substituting the value of F_u = 1,141 lb per shear plane at δ = 0.225 inch (table 5) and the value of nail yield moment, M_y = 178.3 lb inch (table 3) gives

$$f_1 = 2{,}050 \text{ lb/in. (at } \eta_1 = 0.225 \text{ in.)}$$

The theoretical equation fitted to test group 17 is thus

$$F_{uo} = 2{,}750\ (\delta)^{1/8}$$

and the derived wood embedding relationship

$$f = 2{,}050\left(\frac{\delta}{\eta}\right)^{1/4}$$

From figure 19 we see that the theoretical curve fitted to the data coincides with the actual load-deformation curve. The derived f_1 value can then be used to predict the load-deformation relationship for wood-to-wood joint, test group 1. At joint deformation, δ = 0.225 in., $\eta = 0.5\delta$ for each wood member.

$$f = 2{,}050\left(\frac{.113}{.225}\right)^{1/4} = 1{,}730 \text{ lb/in.}$$

We include a small adjustment for nail diameter (table 3)

$$1{,}730\ \frac{.225}{.232} = 1{,}680 \text{ lb/in.}$$

The ultimate load for a single shear wood-to-wood joint (Aune 1966) is of the form

$$F_{uo} = \frac{2\sqrt{2}}{3}\sqrt{2fM_y}\left(\frac{\delta}{2\eta}\right)^{1/8}$$

$$F_{uo} = 880\ (\delta)^{1/8}$$

Where joint deformation δ consists of embedding η in both the side and center members, $\delta = 2\eta$.

Again the predicted curve coincides with the actual test data, joint deformation for values above 0.12-inch (fig. 19).

This method may prove an alternative to the current way of computing a nail joint's load-deformation behavior.

Acknowledgment

The senior author was supported by the Norwegian Institute of Technology at the University of Trondheim, and the Norwegian Council for Scientific and Industrial Research. The research was performed at the Forest Products Laboratory, Forest Service, U.S. Department of Agriculture.

List of Symbols

a, x, x_1 = distance along the nail embedding the wood (in.)
e = insulation thickness, gap (in.)
f_1 = wood embedding stress at deformation η_1 (lb/in.)
f_e = wood embedding strength (lb/in.)
f_y = yield strength of the nail (lb/in.^{2})
F = lateral force on nail (lb)
F_u = computed yield load of joint assuming plastic wood embedment (lb)
F_{uo} = computed yield load of joint assuming fourth-root wood embedment (lb)
ℓ = nail length (in.)
M_y = yielding moment of the nail (lb in.)
t_1, t_2 = wood member thickness (in.)
W = work done by joint (lb in.)
α = t_2/t_1, ratio of *main* member thickness to *side* member thickness
β = ratio of joint embedding strengths
$\gamma = \frac{M_y}{f_e}$
δ = joint deformation (in.)
η, η_1 = wood embedding deformation (in.)
θ = angle
θ_1 = angular deflection
ξ = coordinate for integration